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ORIGINAL PAPER

The *ZIGZAG*: Contributions of African Mathematical Knowledge on Gold Mines to Develop a Pedagogical Action for Ethnomodelling Through the Elaboration of Ethnomodels

Daniel Clark Orey[1] · Milton Rosa[1] · João Batista Nunes da Silva[2]

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Abstract

The form of a *zigzag* is rarely the shortest distance between two points. Ethnomodelling is often a complex form of doing research and *zigzags* between those who do (insiders, emic) and those who observe (outsiders, etic). Eight mathematics teachers, two gold mine owners, and two gold mine guides, were participants of this study. It is important to value cultural and historical aspects of mathematics, in particular, mathematical practices related to knowledge of enslaved African people, which were used in the construction of gold mines in Vila Rica village (presently Ouro Preto), in the captaincy of Minas Gerais, in colonial Brazil. We visited two gold mines constructed in the 17th century. For data collection and analysis, and interpretation of the results, we used a methodological design adapted from Grounded Theory through the conduction of open and axial codings that enabled the identification of preliminary codes and two conceptual categories. Results showed that ethnomodelling enabled the development of dialogues between school mathematical knowledge and mathematical *knowing* and *doing* of the formerly enslaved African people. This mathematical knowledge helped us to develop pedagogical actions, which enabled participants to understand mathematical knowledge used by the enslaved African people in the construction of gold mines in Brazil.

Keywords Ethnomodelling · Ethnomodels · Gold mines · African mathematical knowledge · Enslaved African people · Grounded theory · Pedagogical action

1 Introduction

As we conducted our investigations within the scope of the academic master's degree in mathematics education, a series of ideas, concepts, theories, and discussions emerged during the data collection phase that caused us to reflect on how mathematical practices and pedagogical actions can be enhanced in classrooms. Thus, we justify a search for the development of pedagogical actions for teachers that connects school mathematics (etic/global) and African mathematical knowledge (emic/local) during the conduction of dialogic (glocal) approaches of ethnomodelling with gold mine owners and guides in local gold mines in Ouro Preto.

This context shows that an important indication of the development of African mathematical knowledge is related to the advancement of geometric knowledge, which was not limited to connections with notions and concepts present in Euclidean geometry because members of African cultures tend to intuitively recognize shapes, such as catenaries, in the external world.

We dedicate this article to the memory of the African people who were kidnapped, enslaved, and involuntarily brought to Minas Gerais during colonial Brazil. As well, we acknowledge the traditional owners of the land where this research occurred and through this work, we pay our respects to the elders, past, and present, and to the many diverse peoples who have sacrificed much to live in this truly remarkable place. Gratidão!

✉ M. Rosa
milton.rosa@ufop.edu.br

D.C. Orey
oreydc@ufop.edu.br

J.B.N. da Silva
joao.nunes@educacao.mg.gov.br

1 Universidade Federal de Ouro Preto (UFOP), Rua Pref. José de Castro, n. 32, apto 101, Bairro Cabeças, Ouro Preto, Minas Gerais, Brasil

2 Universidade Federal de Ouro Preto (UFOP), Rua Rio Piracicaba, n. 120, Bairro Jota, Brumadinho, Minas Gerais, Brasil

In the African context, other mathematical logics of geometric composition, such as fractal geometry, were also developed by members of distinct cultures in that continent. For example, intuitive notions of fractal geometry and its patterns have also been used in traditional African architectural designs, hairstyles, fabrics, sculpture, painting, metalwork, games, quantitative techniques, and religious and symbolic systems for centuries. This knowledge and practices were fundamental to the development of many African knowledge systems.

On the other hand, mathematical knowledge is an integral part of contemporary society because it is influenced by human activity and it is a product of sociocultural nature, as it involves formal and informal mathematical ideas, procedures, techniques, and practices present in the daily activities developed by members of distinct cultures. Studies related to sociocultural constructions of mathematical knowledge (D'Ambrosio, 2006; Madruga, 2022; Rosa & Orey, 2013) led to the conduction of investigations that dealt with processes of ethnomathematics and modelling through ethnomodelling.

In this context, ethnomathematical and sociocultural studies enables the advancement of mathematical knowledge in the school environment and the understanding of mathematical *knowing* and *doing* developed by members of the formerly enslaved African people in the period of the discovery of gold and mine construction over 300 years ago during the Portuguese occupation and colonization of Minas Gerais, Brazil.

Thus, the research question of this study is: *How might dialogue*[1] *between mathematics teachers and local gold mine owners and guides contribute to their understanding of mathematical knowledge used by enslaved African people in the construction of gold mines through the elaboration of ethnomodels?* The specific objectives are to:

a) understand how eight middle and high school mathematics teachers become aware of local mathematical knowledge used by enslaved[2] African people in the construction of gold mines in Ouro Preto during their dialogue with local gold mine guides and mine owners.
b) analyze how local knowledge developed by enslaved African people can be contextualized through the elaboration of ethnomodels to understand school mathematical knowledge through the development of a pedagogical action of the dialogic approach of ethnomodelling.

In this context, ethnomodelling is a translational process between locally developed mathematical *knowing* and *doing* and school mathematical knowledge through dialogue and its use in the pedagogical actions in the classroom (Rosa & Orey, 2017). As well, dialogues between s mathematics teachers and the gold mine owners and guides show the possibility for the elaboration of emic (local), etic (global), and dialogical (glocal) ethnomodels.

In the ethnomodelling process, ethnomodels provide consistent representations of the mathematical knowledge socially constructed and shared by members of the formerly enslaved African people by linking the development of school mathematical practices with their cultural heritage by adding the cultural aspects of mathematics to the modelling process.

2 Theoretical Background

This section is composed by three sub-sections described below.

2.1 Brief History of Slavery in Brazil

The Atlantic slave trade began in 1444, when Portuguese traders brought the first enslaved people from Africa to Europe. Portugal, later, in the 16th century, was the first nation to buy enslaved people from West Africa and transport them across the Atlantic Ocean. In 1526, Portuguese traders carried the first shipload of enslaved African people to Brazil, in the Americas, and other European nations followed them in this inhumane trade (Saunders, 1982). In this context, Klein and Luna (2010) affirm that Brazil's forced labor system was the largest and most continuous of all slave societies in the world and has shaped Brazilian ways of life and culture in complex ways up to the present.

On May 13 1888, Brazil became the last country in the Western world to abolish slavery, and that Portugal was one of the first European empires to use slavery as its main tool for enrichment via colonization. During the second decade of the 16th century, the first ships arrived in Brazil bringing enslaved people, mainly from Central Africa and, this continued until 1850, when the transatlantic transfer of captives to Brazil was prohibited. Between these two dates, 4.8 million enslaved people were sent to Brazil to work on plantations, gold and diamond mines, and home services (Hébrard, 2013).

In the state of Minas Gerais, there is a large contingent of Afro-Brazilians that compose its population, and this is due to the process of slavery that once took place in this

[1]Dialogue is about deepening understanding as part of the process of making a difference in the world. It is a co-operative activity involving respect, which is a process that can be seen as enhancing community and building social capital and to leading us to act in ways that make for justice and humanity flourishing (Freire, 1972).

[2]We preferred to use the term *enslaved* instead of the word *slave* to describe African people held in Brazilian slavery possession and to acknowledge the horror and exploitation they were forced to face during their lives. The word *slave* denotes the inherent identity of a person or people treated as chattel or property, while the term enslaved African people emphasizes the slave status that was imposed on them as possessions.

region. The practice of gold mining occurred, mainly, in the region of Vila Rica (Ouro Preto), as gold exploration was the main activity of the economy that held the largest enslaved contingent in the Captaincy of Minas Gerais (Silva, 2023).

At the height of gold exploration in this captaincy, slave labor was essential for the exploration of this ore. Because of their mathematical knowledge regarding to the development of the gold mining processes in Africa. Thus, Gomes (2021) states that it is estimated that, in the 18th century, around 600,000 enslaved African people were involved in gold and diamond mining in the captaincy of Minas Gerais, which represented 20% of the total number of African captives brought to Brazil during this period.

Similarly, Silva (2023) showed that contributions of enslaved African people from this region is related to the technical, scientific, and mathematical improvement of gold mining, which was important for the economic development of colonial Brazil in the 18th century. Africans originally from the Gold Coast were more valued for being knowledgeable and experienced miners, with many of these enslaved African people being prominent goldsmiths and blacksmiths.

According to Marie (2015), enslaved people from Africa brought engineering knowledge to develop the mining work, including construction of the gold mines. After the first peak of mining in Brazil, which was alluvial, carried out in the river, it was necessary to use specialized work for underground mining. Thus, enslaved African people were seen as holders of scientific knowledge and mathematical techniques used to construct these mines.

2.2 Ethnomodelling

The study of ethnomodelling is related to communicative, analytical, and material/technological instruments that are developed by members of distinct cultures. Mathematical ideas, procedures, techniques, and practices (tics) developed by members of distinct cultural groups arise from an inseparable set of techniques for dealing with the environment (mathema) in a constant search to understand the facts and phenomena that occur in their own sociocultural context (ethno) (Rosa & Orey, 2013).

According to D'Ambrosio (2006), mathematical knowledge was accumulated throughout the history of humanity and contextualized in the daily lives of members of distinct cultures, so that they could solve problems, and situations faced in their lives driving the evolution of mathematical practices. Members of each cultural group develop their own locus and corresponding abilities that enable them to interpret and optimize their own mathematical knowledge (Rosa, 2010). This includes the incorporation of Western mathematics in unique and interesting blends trough the historical and sociocultural perspective of mathematical modelling.

In this context, Kaiser and Sriraman (2006) emphasized the role of mathematics in society through the development of sociocritical and sociocultural modelling perspectives. For example, Yang et al. (2022) discussed sociocultural factors that "influence the implementation of mathematical modelling education such as differences of theoretical perspectives of modelling or ways of teaching" (p. 155). They also discussed the "approach of ethnomodelling to expand the understanding of social and cultural influence on mathematical modelling education" (p. 155).

Ethnomodelling aims to develop pedagogical actions whose epistemological assumptions are linked with a broad historiography that starts from the reality and background of the learners, and which seeks to value and respect their historical knowledge acquisition through a cognitive approach with an in-depth cultural foundation. This general view of mathematics from a cultural perspective is one of the main goals of ethnomodelling (Rosa & Orey, 2021).

Similarly, ethnomodelling enables the development of an awareness and appreciation of mathematical knowledge produced in distinct contexts, as well as respecting its local meanings (emic) in the educational system (global) through the development of pedagogical actions that bring schools closer to the members of the communities they serve (glocal) by adding mathematical practices developed locally through *cultural dynamism*[3] (Madruga, 2022). This context enabled Yang et al. (2022) to state that:

> (…) mathematical modelling is a socio-cultural construct, and that various approaches can be identified. Cultural and socio-cultural differences regarding mathematical modelling can be related to – but certainly do not completely coincide with – cultural and socio-cultural differences in the teaching and learning of mathematics more generally (p. 170).

In this socioculturally oriented research perspective, Kaiser and Schukajlow (2022) emphasized the "cultural and socio-cultural influences on the implementation of mathematical modelling in classrooms and consequences for mathematical modelling education research including the ethnomathematical perspective" (p. 147) through ethnomodelling.

2.3 Emic (Local), Etic (Global), and Dialogic (Glocal) Ethnomodels

The theoretical foundation of ethnomodelling fosters the methodological potential for the development of pedagogi-

[3]The connection and exchange between both school and local knowledge can be strengthened by the process of knowing and doing that is gained and supported by cultural dynamics, which occurs when members of distinct cultures encounter, produce, generate, organize, disseminate, and institutionalize mathematical knowledge (Rosa & Orey, 2021).

cal actions in classrooms, which enables the development of connections between cultures and mathematics when seeking the adoption of innovative experiences for the modelling processes through the elaboration of ethnomodels.

Researchers are able to highlight how etic ethnomodels focus on the analysis of mathematical ideas, procedures, techniques, and practices can be shared between cultural groups by using common definitions and metric categories. The emic ethnomodels are based on characteristics found in mathematical knowledge systems present in the lives of members of distinct cultures whose practices have been internally ethnomodeled (Rosa & Orey, 2017).

We emphasize that systems are sets of interacting and interdependent components that, together, form a unitary whole with specific objectives and functions, which are composed of elements present in reality. The analyses of the interrelation among these elements develop reflection, criticality, understanding, and comprehension of phenomena that are parts of the daily life of members of distinct cultures. These components are related to social, cultural, economic, political, and environmental contexts (D'Ambrosio, 1985).

In this context, mathematical systems are used by members of distinct cultural groups, who apply ideas, procedures, techniques, and everyday practices, including mathematics, which help them to solve problems faced in their own reality through the elaboration of ethnomodels. Research related to the application of local "mathematics to solve real-world problems, in the broad sense (. . .), is often called applying mathematics, and a real-world problem that has been addressed utilizing mathematics is called an application of mathematics. Sometimes, though, the notions of 'applying' or 'application' are used for any kind of linking of real world and mathematics" (Niss et al., 2007, p. 10).

In the ethnomodelling process, glocalization (dialogic) promotes the development of a theoretical basis for the incorporation of mathematical knowledge systems arising from local cultural practices (emic) by connecting them with resulting multiple (global) world views (Orey & Rosa, 2021). For example, globalized mathematical procedures and practices often emerge from localized mathematical traditions, customs, techniques, ideas, and notions through the development of cultural dynamism and dialogue.

The concept of dialogue is triggered by recognizing the coexistence of diverse paradigms in the same system, which are often considered opposites. Yet, they are complementary, and they can be integrated into the same phenomenon (Morin, 1977). Thus, complementarity exists between mathematical ideas, techniques, procedures, and practices developed locally (emically) and globally (etically) through the development of cultural dynamism (Rosa & Orey, 2013).

However, this dialogue depends on *acts of translation* between local (emic) and global (etic) approaches (Orey & Rosa, 2021), which describe for members of one cultural community how members of another interpret the world and their place in it. They also refer to different forms of negotiation that members of distinct cultures engage in when they are displaced from one cultural community into another (Buden & Nowotny, 2009). This particular translational process involves a dynamic dialogue between two distinct cultural systems through which grow a need to understand how local mathematical ideas, procedures, techniques, and practices, are connected to global realities.

3 Methodological Procedures

To answer the research question of this study, a qualitative study based on an adaptation of Grounded theory (Glaser & Strauss, 1967) was conducted with the participation of mathematics teachers, gold mine owners, and gold mine guides. This study was conducted in two gold mines in Ouro Preto to find information to show how ethnomodelling enabled the development of dialogues between school mathematics and the mathematical knowledge of the enslaved African people in Brazil.

Grounded Theory presents us with an inductive, qualitative, and exploratory methodology in which the main objective is to apply detailed analysis and an in-depth interpretation of data through encodings, which enables the identification of concepts based on data analysis (Glaser & Strauss, 1967). In this study, data were collected, analyzed, and coded according to the procedures used in an adaptation of the Grounded Theory, which assisted us in the conduction of the analytical and interpretative phases of this investigation.

In this adaptation, selective coding, identification of a central category, and the development of an emerging theory from the collected data, were not used as a methodological procedure, given that our primary goal was to answer the proposed research question. Participants of this study were 8 mathematics teachers who teach middle school mathematics. Figure 1 shows some general information related to these participants.

Participants mathematics teachers teach in two different schools located in two towns in the metropolitan region of Belo Horizonte, Minas Gerais, Brazil. They live and work approximately 110 km away from Ouro Preto, where they visited the two gold mines. These participants are external to the former slave cultural system because they are working in another cultural context, which is the school environment.

It is important to state that 2 gold mine owners and 2 gold mine guides were also participants of this study. These professionals are residents of Ouro Preto. Data analysis show that three of these guides are Afro-Brazilian with predominantly sub-Saharan African ancestry, and descendants of enslaved African people who were forced to work in diverse activities in Brazil such as the now abandoned gold

Fig. 1 General information about mathematics teachers. Source: Authors' personal file

Participants	*Gender*	*Race*	*Age*	*Education*
Teacher A	Male	Black	50	Bachelor's Degree in Mathematics and Specialist in Mathematics Education
Teacher B	Female	White	36	Bachelor's Degree in Mathematics
Teacher C	Female	White	40	Bachelor's Degree in Mathematics
Teacher D	Male	White	36	Bachelor's Degree in Mathematics
Teacher E	Female	Black	52	Bachelor's Degree in Mathematics and Specialist in Mathematics Education
Teacher F	Female	Black	27	Bachelor's Degree in Mathematics
Teacher G	Male	White	34	Bachelor's Degree in Mathematics and Specialist in Mathematics Education
Teacher H	Female	Black	58	Bachelor's Degree in Mathematics and Specialist in Mathematics Education

Fig. 2 General information about gold mine owners and guides. Source: Authors' personal file

Participants	*Gender*	*Race*	*Age*	*Education*
Guide A	Male	Black	40	Philosophy
Guide B	Male	White	30	History
Owner A	Male	Black	47	Civil Engineering and Geology
Owner B	Male	Black	42	Geology

mines and which serve as tourist attractions. Thus, three of these professionals are considered members of the cultural group (internal) of the formerly Brazilian slave system and external to the mathematical school context, yet, through dialogue, they developed and use a holistic understanding of both mathematical systems. Figure 2 shows some general information related to gold mine owners and guides.

Typically, guides develop relevant cultural, historical, and practical knowledge that they share with the people they guide. They also possess knowledge, competencies, and skills about their chosen place and/or area to work. Data analysis also shows how guides in this study possess substantial knowledge about gold mines because they had a personal interest in the general knowledge that enslaved African people brought to Brazil. Thus, *Guide A* commented that:

> My knowledge about the construction of gold mines was passed to the members of my family from generation to generation by my ancestors and I also developed some research on this subject. In my investigations, I found some reports about the way gold mines were built and the shape of curves, so as not to collapse because they are more resistant.

Continuing with this analysis, *Owner B* stated that: "In researching my ancestors, I discovered that in the specific case of mining activity, the African miners who came to Brazil as enslaved people brought important contributions of mathematical and scientific knowledge to the construction of the gold mines".

In this context, Hébrard (2013) stated that experiences and mathematical knowledge African enslaved people brought from their homeland became indispensable elements in constructing gold mines at the beginning of the gold cycle in Minas Gerais. They also introduced their own mining methods to extract gold from the mines.

With regard to the mathematical knowledge of enslaved African people, *Guide A* affirmed that: "We are descendants of the enslaved people who helped to build Brazilian society and identity, which without the knowledge of our ancestors this country would not exist the way it exists today. If Brazil has wealth and is advanced, it is thanks to our efforts", while *Guide B* highlighted that:

> The greatest heritage of Ouro Preto is related to the extraction of gold because all the technology and mathematical and scientific knowledge that were used for the construction of the mines, as well as in the work that accompanied the mining was the domain of the enslaved people.

As for the historical knowledge of the African presence in mining, gold mine owners and guides affirmed that they did not have access to these narratives as students of primary, secondary, and higher education. They also said that knowledge about enslaved people miners was acquired through their family members and investigations they developed after graduating at a higher level.

3.1 Results from the Visit to DuVeloso and Jeje Gold Mines

The reason for choosing the DuVeloso and Jeje gold mines for visitation was influenced by the work conducted on these tourist sites, which are considered as spaces of mining heritages that are representative of African scientific, technological, and mathematical knowledge. These mines are also used as Afro-centered historical narratives that present and interpret these spaces as decolonized historical products, in addition to the fact that these cultural artifacts are open for tourist visitation.

The first visit happened on July 16th, 2022, when the owner of DuVeloso gold mine received mathematics teachers accompanied by the researchers. Next, all participants

Fig. 3 DuVeloso gold mine owner and mathematics teachers. Source: Authors' personal file

Fig. 4 Visit to the Jeje gold mine. Source: Authors' personal file

were instructed in the use of safety equipment when entering the mine, such as a bonnet, helmet, and mask to prevent COVID-19. The main objective of this visit was to identify the mathematical knowledge and/or practices used by the enslaved African people in the construction of gold mines in Ouro Preto.

Before they started the visitation, mathematics teachers had a brief historical introduction about this gold mine and their construction and operation by the enslaved African people during the gold cycle. In his introduction, *Owner A* explained about mine's historical, geographic, and topographic features, including the labor and survival practices to extract gold. He also commented that:

> Enslaved African people were captured and brought to Brazil. They were robbed and kidnapped from their land, but they also brought strategies and techniques to construct and work in the mines. They were intelligent and very strong.

In this regard, Calógeras (2018) highlights that there was a predominance of slave labor specialized in the exploration and construction of gold mines during the gold cycle in colonial Brazil. And for their own survival, it was up to enslaved Africans themselves to transfer their knowledge in mining to Portuguese America.

After this introduction, participants began to visit the mine under the guidance of *Guide A*. Figure 3 shows the owner of DuVeloso gold mine explaining to mathematics teachers about its construction and operation.

Inside of the gold mine, *Guide A* commented that:

> Historians argue that it is from the enslaved African people that the Portuguese based themselves on their mathematical knowledge to be able to mine here in Minas Gerais in the diaspora time in the 18th and 19th centuries. So, there is a consensus that Africans brought to Brazil a lot of mathematical and scientific knowledge.

Continuing with his explanation, this guide also stated that: "Mathematical knowledge and practices that came from Africa does not come from academia and schools. It came with the enslaved African people who too developed this kind of *knowing* and *doing*". In this context, Gomes (2021) argues that:

> Among all enslaved Africans people with specializations, those who appeared most frequently in the documents of the slavery trade and in the records of slavery owners in Brazil and other parts of America, especially, at the gold rush, were those called *miners*, a generic designation of different peoples and ethnicities who were living in the Gold Cost, Africa, who developed scientific and mathematical knowledge (p. 85).

In relation to the tunnels of this mine, *Guide A* commented that: "The way in which the tunnels inside of the gold mines were made is unquestionable. It was necessary to have some mastery of engineering and mathematics for the calculations needed for its structures to remain sustained to this day".

Results showed that enslaved African people developed scientific and mathematical knowledge that helped them with the construction of gold mines and its production. Thus, Gomes (2021) affirms that African born mathematicians and engineers were brought to Brazil by the Portuguese owners as enslaved people in order to apply their mathematical knowledge in the construction of the mines.

Upon finishing their visit to this gold mine, at 10.20 am, participants visited Gold Jeje gold mine and arrived there at 11.10 am (Figure 4). Participants mathematics teachers were received by the gold mine owner. However, before entering the mine facilities, these participants were properly instructed to protect themselves from possible physical harm by wearing safety equipments as previously described. Then, *Owner B* gave an introduction about the visit to the mine by highlighting the specific aspects related to its construction and operation. After this brief introduction, participants visited the mine under the guidance of *Guide B*.

According to this context, *Guide B* commented that the "Enslaved African people were chosen and valued for their work capabilities, they developed an extensive

Fig. 5 Excerpt of the dialogue between *Teacher A* and *Guide B* regarding the structure of Jeje gold mine. Source: Authors' personal file

Guide B: Basically, the tunnel has a horseshoe shape.
Teacher A: Can you repeat this information about the horseshoe shape of the tunnel?
Guide B: I believe that the horseshoe shape is due to the transfer of weight. Weight transfer basically takes the weight from the ceiling, transfers it to the walls and falls into the horseshoe, where the horseshoe is a little more closed.
Teacher A: In my opinion, this is the concept of catenary that you see throughout this tunnel. The catenary has this shape that is a mathematical instrument in which the gravity comes over this curve. Thus, gravity distributes the weight equally across the entire catenary by reaching its base. This one is an inverted catenary.
Guide B: Does it work upside down?
Teacher A: Exactly, those curves you see on the bridges are catenaries. That's why I would like you to repeat what you said because it is an informal explanation about catenary. And we are commenting about this curve on the way it is done in schools, which has a similar explanation to what you explained.
Guide B: Really?
Teacher A: Yes, you just explained this concept of yours, the way you observe these curves in your work, the way it's passed from one to the other, the way you learned it. This is the way you observe in your culture.
Guide B: There's just something I wanted to know about how it works in here. You notice at the time that it does it like this in a *zigzag* shape.
Teacher A: Wouldn't it be a matter of point of infinity? Because if you go straight into this tunnel, you can see it and you can make it difficult to escape by *zigzagging*. So, what do you think it would be?
Guide B: In my opinion it's a weight distribution principle or something like that. And, in that part there you create a return.
Teacher A: There is a story called latent, which has no scientific explanation, right? Now, the cultural explanation would be that some African people said that evil walks in a straight line, so for you to divert evil you must walk in a zigzag because evil gets lost along that path. So, this is a cultural explanation that is latent and there is no scientific explanation for it.

knowledge of the art of mining, estimation, calculations, and the application of scientific and mathematical knowledge". Based on this, *Teacher F* commented that: "What became more evident was how the enslaved people had knowledge in multiple areas of the mining process, including the steelmaking part of making tools and the use of mathematical knowledge". Thus, *Owner B* highlighted that:

> The enslaved African people dug mines in the shape of X shape, and it actually made a lot of sense when we see it in practice. I also highlight that they were very aware of when they should stop the mining process, as they knew that gold was present in the quartz and when it ran out, the gold was possibly running out as well.

In this regard, D'Ambrosio (2011) stated that culture is: a) a set of shared knowledge and compatible behaviors, b) the accumulation of knowledge shared by members of a given cultural group results in the compatibility of their behavior, and c) after being accumulated and disseminated they constitute the culture of that group. Participating mathematics teachers and *Guide B* sharded a combination of local (emic) and school (etic) mathematical knowledge that evidenced the beginning process of mathematization and further use of ethnomodelling. Figure 5 shows an excerpt of the dialogue between *Teacher A* and *Guide B* regarding the structure of Jeje gold mine.

In this context, the *zigzag*[4] shape was used as a form of making sure that sure that any escape from the mine was harrowing, as well as any unauthorized removal of gold from the gold mines. These procedures were connected to the African people's cultural and spiritual traditions because the *zigzag* pattern emerged as a metaphor for the development of their mathematical knowledge. These procedures were connected with the African people's cultural and spiritual traditions because the *zigzag* pattern emerged as a metaphor for the development of their mathematical knowledge.

Regarding to the structure of the gold mine's construction through mathematical knowledge passed from generation to generation, *Guide B* mentioned that "some tunnels in the interior of the mine possess a *horseshoe*[5] shape" (Fig. 6).

After discussions between mathematics teachers and *Guide B* about the curves having a parabolic or catenary shape, *Teacher A* commented that:

[4]According to Thompson (1983), there was a belief shared by many African enslaved people that was diffused independently in both mines that evil travels in straight lines. In this context, African patterns may be used in several ways to stop the flow of evil, such as by breaking the path into *zigzag* pathways. Thus, to throw off *demon spirits*, they could not apply a perfectly lined pathway in or out of the mines. Instead, the construction pattern had to incorporate this design in many 18th century mine tunnel constructions in Ouro Preto.

[5]It is important to consider that a horseshoe shape does not approach a catenary function. We highlight that different functions have similar forms, such as $f(x) = x^2$, $f(x) = x^4$, and $f(x) = \cosh x$, but, that similarity is not a criterion to enable us to state that they are catenaries because only coshx is a catenary.

Fig. 6 Horseshoe shape of the tunnel in the gold mine. Source: Authors' personal file

> Because the tunnel of this mine is narrower, it is possible to measure the curves and even the support arches, which, in turn, would allow us to check whether these arches are parabolic or catenary. However, for now it is expected that they are catenary, but to confirm this hypothesis it would be necessary to conduct a modelling work.

Similarly, *Teacher D* argued that: "The curves at the top inside the mines can be initially associated with parabolas, but with the support knowledge, they appear to have a catenary shape". In this regard, Cvetković et al. (2021) stated that the catenary as an optimal architectural shape has been known since ancient times. It is a curve that has become one of the most common shapes used by humanity, not only in architecture but also in the field of construction and engineering.

This discussion enabled mathematics teachers to reach a consensus in which the arch (emic/local view) shape of some of the curves in the tunnels of the mine is close to the geometric shape of a catenary (etic/global view), which can be translated into a dialogic ethnomodel (glocal/cultural dynamism). Figure 7 shows a dialogic ethnomodel that can represent a catenary function in the tunnel of Jeje gold mine.

The proposed ethnomodel was elaborated by using a photograph of the tunnel in the gold mine and GeoGebra software. However, this ethnomodel could be better described by using the current tunnel measurements of the curvature of the gold mine. Figure 8 shows an etic ethnomodel that can represent the tunnel of Jeje gold mine as proposed by *Teacher B*.

This ethnomodel shows the relation between school mathematics (etic) and local (emic) mathematical knowledge applied in the construction of the tunnel in this gold mine. These results show that both emic and etic approaches were combined to give a holistic view of the object of this study. Thus, Orey and Cortes (2022) state that knowing, understanding, and explaining an etic (global) ethnomodel offers an opportunity to comprehend mathematical knowledge developed by members of distinct cultures (emic/local) through dialogue.

Consequently, the relevance of glocal approach is related to the development of cultural dynamism through an understanding of the evolution of emic (local) mathematical knowledge and its connection with mathematical knowledge studied in the schools by highlighting their complementarity through descriptive translations (Rosa & Orey, 2017).

At the end of this visit, mathematics teachers also partook in a brief explanation about the amount of gold removed from the mines in this region during the gold cycle. As well, *Owner B* explained about the amount of money collected from taxes and also regarding behavioral persecution and suffering of enslaved Africans in the gold mining sites. Figure 9 shows the explanation given by *Owner B* after visiting Jeje gold mine.

The visit to this mine ended at 12.40 pm. After these two visits, mathematics teachers discussed what they learned from the mathematical procedures, techniques, and practices regarding to the construction of gold mines. For example, *Teacher B* commented that: "The curves in these gold mines tunnels have the shape of a catenary arch", while *Teacher E* highlighted that: "The catenary shape is considered an ideal support for distributing the weigh along the curve to support structures of a building in accordance with assumptions of mathematics and engineering of the time".

Continuing with this discussion, *Teacher F* commented that: "The curves of the tunnel are similar to an inverted catenary, which is a curve that can be formed by a string attached at its ends. It is a subject to the action of gravity", while *Teacher D* affirmed that: "This technique was used in numerous mines by enslaved African people, which shows that they brought mathematical and engineering knowledge here to construct the mines".

Similarly, *Teacher C* emphasized that: "In my point of view, this catenary curve shape is the most correct for an arch to support its own weight throughout the tunnels, and maybe the enslaved African people brought this knowledge to Minas Gerais", while *Teacher B* stated that: "It appears that, in the tunnels, the arches describe a set of curves that can be generated by a rope suspended from its ends and subject to the action of gravity".

In this regard, Gerdes (2002) stated that many African cultures produced scientific and mathematical expertise, and the *Africanization* of knowledge can be understood as an attempt to understand, analyze, and disseminate ideas produced by members of distinct cultures originating in the African continent. The dissemination of such mathematical knowledge may involve the incorporation of these ideas into education today and in the future. Accordingly, Rosa (2010) commented that the diffusion of local knowledge is significant in the spread of specific mathematical techniques for solving problems faced by members of distinct cultural groups in their daily lives.

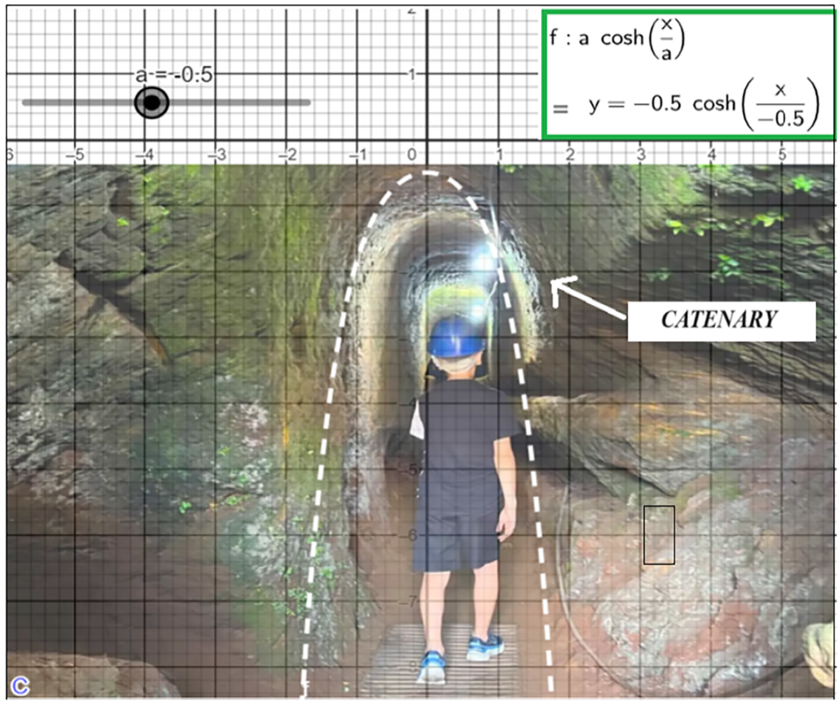

Fig. 7 Dialogic ethnomodel of a catenary function in the tunnel of Jeje gold mine. Source: Authors' personal file

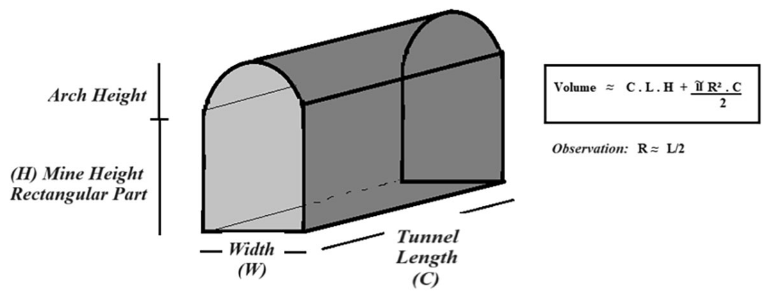

Fig. 8 Etic ethnomodel proposed by *Teacher B*. Source: Authors' personal file

Fig. 9 Explanation given by *Owner B* after visiting Jeje gold mine. Source: Authors' personal file

On the other hand, *teacher F* argued that: "Training moments like this one contributed to the improvement of our mathematical understanding by improving our teaching practices inspired by experiences of enslaved African people". Thus, training moments are related to any environment, such as gold mines, which allows for exchanging knowledge, perspectives, strategies, concepts, and practices. These moments promoted a constant movement toward learning and the development of their teaching practices. Thus, pedagogical actions can be improved by using mathematical knowledge developed by the enslaved African people.

After finishing these visits, in order to start the analytical process, we used direct quotations from the participants that were related to the open and axial codings, which helped us to identify preliminary codes and conceptual categories during the conduction of analytical and interpretative phases of this study. This process of conceptual abstraction was developed by assigning concepts (codes) to data (direct quotations), which were related to the dialogues between mathematics teachers and gold mine owners and guides.

Thus, direct quotations were fragmented into smaller parts and were analysed line by line and/or sentence by sentence. This methodological procedure enabled the identification of preliminary codes through common characteristics and concepts obtained in the collected data from the answers given to questions from the questionnaires and interviews in the open coding process (Fig. 10).

Collected data	Open Coding (Preliminary Codes)
Teacher C: Historians argue that it is from the enslaved African People (1) that Portuguese people based themselves (2) on their mathematical knowledge (3) to be able to mine here in Minas Gerais and in Brazil (2) in the decade of the diaspora in the 18^{th} and 19^{th} centuries (1). So, there is this consensus that Africans brought (4) a lot of this mathematical knowledge to the mines (5).	(1) Slavery history
Teacher F: What became more evident was how the enslaved people (6) had knowledge in multiple areas of the mining process (5), including the steelmaking part of making tools (5) and the use of mathematical knowledge (3).	(2) Diffusion of local mathematical knowledge
Guide B: Basically, the tunnel has a horseshoe shape (3).	
Teacher A: Can you repeat this information about the horseshoe shape of the tunnel? (7).	
Guide B: I believe that the horseshoe shape (8) is due to the transfer of weight (9). Weight transfer basically takes the weight from the ceiling (9), transfers it to the walls (9) and falls into the horseshoe (8), where the horseshoe (8) is a little more closed (9).	(3) Local mathematical Knowledge (emic)
Teacher A: This is the concept of catenary (9) that you see throughout this tunnel. The catenary has this shape that is a mathematical instrument in which the gravity comes over this curve. Thus, gravity distributes the weight equally across the entire catenary by reaching its base (7). This one is an inverted catenary (7).	(4) Cultural Valorization
Guide B: Does it work upside down? (9).	(5) Mining knowledge
Teacher A: Exactly, those curves you see on the bridges are catenaries (9). That's why I would like you to repeat what you said is an informal explanation (7) about catenary (9). And we are commenting about this curve (9) on the way it is done in schools and academia (10), which has a similar explanation to what you explained (3).	(6) Enslaved African people
Guide B: Really? (11).	
Teacher A: Yes, you just explained this concept of yours (3), the way you observe these horseshoes in your work (8), the way it's passed from one to the other (11), the way you learned it (3). This is the way you observe in your culture (12).	(7) Interested in knowing local mathematical knowledge
Guide B: There's just something I wanted to know about how it works in here (11). You notice at the time that it does it like (3) this in a *zigzag* shape (9).	
Teacher A: Wouldn't it be a matter of point of infinity? (9). Because if you go straight in this tunnel, you can see it (3) and you can make it difficult to escape by *zigzagging* (8). So, what do you think it would be? (11).	(8) Local jargons
Guide B: I believe it's a weight distribution principle or something like that (9). And, in that part there you create a return (3).	
Teacher A: there is a story called latent (1), which is a story that has no scientific explanation, right? (3). Now, the cultural explanation would be that some African people (12), for example, said that evil walks in a straight line (3), so for you to divert evil you must walk in a zigzag because evil gets lost along that path (3). So, this is a cultural explanation (12) and a latent thing and there is no scientific explanation (3).	(9) Notions of catenary
Teacher B: The tunnel of this mine is very narrow (3), it would be easy to take measurements (13), which allow for optimal	(10) Global mathematical knowledge (etic)

Fig. 10 Preliminary codes identified in the open coding process. Source: Authors' personal file

approximations of volume of rock extracted from the mine (14).	
Teacher B: The curves in these gold mines tunnels (3) have the shape of a catenary arch (9).	(11) Transcendence of knowledge.
Teacher E: The catenary shape (9) is considered ideal for distributing the weigh along the curve (3) to support structures of a building (10) in accordance with the assumptions of Mathematics and Engineering (15).	(12) Cultural Identity
Teacher F: The curve of the gold mine tunnel (3) is similar to an inverted catenary (9), that is, a curve formed by a wire attached at its ends (3), which is subject to the action of gravity (9).	
Teacher D: I think that this technique was used in the mines (3) by enslaved people (6) shows that they brought advanced mathematical and engineering knowledge here to construct the mines (15).	(13) Relation between Mathematics and Culture
Teacher C: In my point of view, this catenary curve shape is the most correct for an arch to support its own weight (9) throughout the tunnels (3), and maybe the African enslaved people (6) brought this knowledge to Minas Gerais (15).	(14) Mathematization Process
Teacher B: It appears that in mine tunnels the catenary curve may describe a set of curves (9) that can be generated by a rope suspended from its ends and subject to the action of gravity (3).	
Teacher C: In my point of view, this catenary curve (9) shape is the most correct for an arch to support its own weight throughout the tunnels (3), and maybe the African enslaved people (6) brought this knowledge to Minas Gerais (15).	(15) Dialogic relation
Teacher F: These formative moments contribute (16) to our mathematics teaching by improving pedagogical practices (17) inspired by lived experiences of enslaved people (13).	(16) Teacher education
Owner of Mine B highlighted that: As I mentioned, the enslaved African people (6) dug mines in the shape of X shape and it actually makes a lot of sense when we see it in practice (5). I also highlight that they were very aware of when they should stop the mining process (5), as they knew that gold was present in the quartz and when it ran out (7), the gold was possibly running out as well (5).	(17) Pedagogical action of ethnomodelling
Guide A: when enslaved African people (6) were captured and brough to Brazil (1) they were searching for their ancestrally (18). And, yes, they were robbed and kidnapped from their land (1), but they also brought an advanced mathematical knowledge (3) to construct and work in the mine (5). They are intelligent and that is very strong (6).	(18) Importance of ancestry
Teacher C: Historians argue that it is from the enslaved African People (1) that Portuguese people based themselves on their mathematical knowledge to be able to mine here in Minas Gerais and in Brazil (15) in the decade of the diaspora in the 18th and 19th centuries (1). There is this consensus that Africans (6) brought a lot of mathematical knowledge (2).	(19) Enslaved people working capabilities
Guide B: The enslaved African people (6) were chosen for their work capabilities (19), they developed an extensive knowledge of the art of mining (5), estimation (19), calculations (19), the application of scientific and mathematical knowledge, among others (19). This comment helped me understand that they had much greater knowledge than we imagined (11).	(20) Enslaved people mathematical knowledge

Fig. 10 (*Continued*)

This analytical procedure insured that the results obtained in this study were interpreted through the elaboration of conceptual categories identified in an axial coding method. Previously, preliminary codes were grouped together according to similar properties and concepts, which helped us to identify the conceptual categories (Fig. 11).

A data triangulation approach was also applied to achieve validity and credibility of the results as well to mitigate the presence of any research biases by using multiple sources of data to examine the proposed problem statement. We gathered information from cross-verifying multiple sources such as interviews, questionnaires, focus groups, mine observations, and three activity modules in order to comprehensively answer the research question.

4 Interpretation and Discussion of the Results

In this study, preliminary codes were identified in the open coding process, and then they were systematized in two conceptual categories identified in the axial coding process: 1) *Local Mathematical African Knowledge: Emic Approach* and 2) *Glocal Dialogic Approach: Cultural Dynamism of Ethnomodelling*. The interpretive process of this investigation was conducted through a description of conceptual categories identified in the analytical process. For the elaboration of these categories, direct quotations from the *mathematics teachers*, *gold mine guides*, and *gold mine owners*, were used to provide a trustworthy context of the problem statement, thus enabling the interpretation of the results obtained in this study.

Fig. 11 Conceptual categories identified in the axial coding process. Source: Authors' personal file

Open Coding (Preliminary Codes)	Axial Coding (Conceptual Categories)
(1) Slavery history (2) Diffusion of local mathematical knowledge (3) Local mathematical Knowledge (emic) (4) Cultural Valorization (6) Enslaved African people (8) Local jargons (11) Transcendence of knowledge. (12) Cultural Identity (18) Importance of ancestry (19) Enslaved people working capabilities	Local Mathematical African Knowledge: Emic
(5) Mining knowledge (7) Interested in knowing local mathematical knowledge (9) Notions of catenary (10) Global mathematical knowledge (etic) (13) Relation between Mathematics and Culture (14) Mathematization Process (15) Dialogic relation (16) Teacher education (17) Pedagogical action of ethnomodelling (20) Enslaved people mathematical knowledge	Glocal Dialogic Approach: Cultural Dynamism of Ethnomodelling

4.1 Local Mathematical African Knowledge: Emic Approach

Interpretation of the results shows that participants mathematics teachers, through dialogues with the mine owners and guides, perceived that enslaved African people, who were forcefully taken from their homes and families in Africa, were responsible for the construction of gold mines during Brazilian colonial times. During the visit to DuVeloso gold mine, *Teacher D* observed that:

> From the knowledge I gained over time on this topic by doing some online investigations to develop pedagogical actions for my students, I understand that it was the enslaved African people who built the gold mines in Ouro Preto by using their mathematical techniques. They developed mathematical knowledge to construct and work in the mines, as well in architecture.

Similarly, *Teacher B* commented that: "enslaved Africans were chosen for their labor and knowledge that was transmitted between generations, such as knowledge of gold veins, the construction of mines and tunnels, and the manufacture of tools used in the construction of mines", while *Teacher F* affirmed that:

> Knowledge of enslaved workers was enormous, in addition to all the culture brought through generations from their original location, which greatly contributed to everything we can see today, in this style of construction of streets, mines, churches and monuments, many of their qualities were suffocated by their owners who oppressed them.

In addition, *Teacher E* affirmed that:

> There is a relation between mathematics and the cultural process of constructing and extracting gold from the mines. These people had knowledge of both everyday mathematics, as well as some knowledge of engineering and chemistry. They used mathematical knowledge of the topology of the mountains to build aqueducts and divert water courses for mining purposes. They also used knowledge about gold density to remove gold from the pans. All this practical and everyday knowledge was embedded in the habits and customs of these people and was fundamental to the mathematical efficiency of their work.
>
> This context allowed *Teacher G* to explain that:
>
> Mathematical knowledge that was passed on by the enslaved African people helped us to understand more completely the thinking and mathematical practices they developed during their lives. They used their knowledge, including mathematics, in diverse activities. So, they were chosen to conduct the work they carried out in the mines and in homes and churches by showing the richness of African culture.

These participants also highlighted some aspects of local mathematical knowledge (emic) that is present in the work practices of African people who were enslaved in Brazil. For example, *Teacher C* added that: "In the contact with the *Owner of Jeje gold mine* what became more evident was how the enslaved African people brought with them and further developed mathematical knowledge regarding mining practices of the time". Thus, *Teacher E* stated that:

> The owner of DuVeloso mine cited a lot about the knowledge of African people who could work in the mine with certain tools to make their work easier.

> The owner of Jeje gold mine recalled the culture and knowledge of African people in relation to the gold mines in Africa needed to run the gold mines in Minas Gerais.

According to Gomes (2021), enslaved African people had unique mathematical and scientific experiences, information, and skills that were of valued and of high interest to the colonizers and, therefore, they also paid different prices for them according to these specializations. The gold in the Brazilian colony was collected through the use of techniques introduced by Africans and which were unknown to most Europeans of the time.

We emphasize the importance of understanding the processes of generation, transmission, and diffusion of previously unacknowledged mathematical knowledge of members of the cultural group of enslaved Africans, with the purpose of using them in maintaining their values, techniques, and mathematical practices. Hence, Martins and Brito (1989) affirm that members of this distinct cultural group developed a technological background in mining, which was diffused, and transmitted orally, from generation to generation, in this process of technological transfer.

This interpretation also shows that *Owner A* and *Guide A* affirmed that enslaved African people developed professional understanding of the mathematical knowledge used in the construction of gold mines that was diffused from generation to generation. This result shows the importance of diffusing mathematical knowledge to show how local practices keep cultures alive. According to Berstein (1965), knowledge of African miners was diffused from person to person, which deals with the development of techniques and strategies necessary for the construction of gold mines.

After the mine visits, *Teacher H* stated that: "I confess that I felt represented, as I am an Afro-Brazilian descendant of enslaved people. The legacy of mathematical knowledge and practices left by them and their descendants in these constructions needs to be valued, appreciated, and respected". This result shows that identification with Black Culture and the mathematical knowledge that enslaved Africans, and their descendants bequeathed are valuable for promoting an appreciation and respect for Afro-descendant culture in Brazil. According to these participants, enslaved African people who worked in the gold mines in Ouro Preto were qualified because they had specialized knowledge to carry out work related to gold mining.

4.2 Glocal Dialogic Approach: Cultural Dynamism of Ethnomodelling

The preliminary codes identified during the open coding composed the conceptual category related to the dialogue between different mathematical knowledge systems through the development of pedagogical actions of ethnomodelling. These actions relate to the application of common day-to-day situations, problems, and phenomena contextualized in classrooms. Activities were selected to be developed with students inside and outside of the school contexts to provide an integrative approach to the mathematics curriculum that considers the connections between local, global, and glocal mathematical knowledge through the elaboration of ethnomodels.

These pedagogical actions seek to understand the traditions, ideologies, cosmovisions, behaviors, beliefs, and mathematical and scientific knowledge of a particular culture by advocating the relevance of, indeed, the complementarity between school mathematics and the local mathematical ideas, procedures, techniques, and practices developed by its members. For example, *Teacher H* stated that:

> (...) this type of pedagogical action enables the development of cultural sensitivity related to knowledge, awareness, and appreciation of mathematical knowledge developed by enslaved people in the construction of gold mines, a connection between knowledge and mathematical practices developed by members of different cultural groups.

Similarly, *Teacher E* highlighted that:

> (...) during the period of slavery, enslaved African people brought mathematical knowledge that they developed in their homeland. Thus, these people used their own mathematics in the construction of mines and in other activities related to gold extraction. Therefore, we must develop pedagogical actions that seek to connect knowledge and practices of local African cultures with the mathematical knowledge taught at school.

In this regard, Rosa and Orey (2017) state that processes of teaching and learning mathematics based on ethnomodelling are intrinsic to the development of pedagogical implications because they enable the investigation of mathematical ideas, procedures, techniques, and practices originating in specific cultural contexts and their complementarity to mathematical knowledge studied in classrooms.

Results show that mathematics teachers commented that connections between school mathematical knowledge and the cultural, historical, social, and architectural features of the mines enable the elaboration of activities that contextualize curricular mathematical content on the mathematical knowledge of formerly enslaved African people in colonial Brazil, which help them to critically reflect and analyze a given contextualized activity.

In this context, Barbosa (2006) discusses the importance of reflecting on the role of mathematics in society by drawing conclusions from studies about the sociocultural dimensions of mathematics, particularly, the critical nature of mathematical models in society. Thus, mathematical models (ethnomodels) are not neutral descriptions about an independent reality, the modelling process has devices that are usually concealed from the general public.

Similar results of a study conducted by Mukhopadhyay and Greer (2001) show how the empowerment of citizens by providing them with the tools, rights, and responsibility to investigate critically and reject mathematical arguments. This enables students to judge applications of mathematics used to describe and analyse diverse aspects of society.

These results also showed the authors that mathematics teachers explained how they might use pedagogical actions of ethnomodelling by applying local (emic) mathematical knowledge developed by enslaved people during the construction of the 18th century gold mines. According to D'Ambrosio (2011), it is necessary for educators to propose classroom activities that originate in the daily lives of students and/or very close to their sociocultural reality to connect mathematics to phenomena that occur in their daily live.

5 Conclusions

Cultures can be seen as a form of molding reality, which determine the development of actions that encourages members of distinct cultural groups to share how they solve problems they face daily. In this context, both approaches, local (emic) and global (etic), are essential for understanding mathematical knowledge that shape ideas, procedures, techniques, and mathematical practices through cultural dynamism.

In this regard, cultural expressions enable a sense of awareness that encourage the development of a sense of cultural dynamism from a different point of view in which mathematics allows us to become allies for promoting dialogue between the school mathematical content (etic/global) and the sociocultural and historical contexts of the students (emic/local) through ethnomodelling.

One of the implications of this study is related to the possibility of developing etic (global), emic (local), and dialogic (glocal) ethnomodels through a holistic understanding of African mathematical knowledge shared among participating mathematics teachers and the gold mine owners and guides in their experiences at the DuVeloso and Jeje gold mines.

Another implication is the development of the dialogical approach of ethnomodelling to promote the connections between emerging and existing mathematics, which enables a sense of mindfulness of this knowledge through the elaboration of mathematization-based activities. It also enables mathematics teachers to perceive the connections of school mathematics with the local history and culture of formerly enslaved African people.

It is recommended the conduction of investigations that deal with the *zigzag* format in many tunnels in gold mines, which were used as a form of making sure no enslaved worker escaped for freedom or left the mine with gold. Another aspect of these future studies is to investigate if the *zigzag* format was connected to cultural and spiritual traditions of the enslaved African people, which became a metaphor for how this particular ethnomodel emerged in this study.

It is also recommended that future investigation in ethnomodelling study the development of mathematical knowledge in order to reclaim the contributions of members of distinct cultures who have been conquered and/or marginalized or who pertain to minority groups, such as the formerly enslaved African people. In this regard, ethnomodelling elicits respect for diverse forms of mathematical knowledge and points a way out of ethical dilemmas brought up by investigations in this area of study.

In conclusion, it is important to state that the influences of African culture and mathematical knowledge on the development of architectural, historical monuments, and mining construction and productions in Vila Rica (Ouro Preto), revealed that mathematical *knowing* and *doing* developed by enslaved African people were both practical and motivating. In this regard, Silva (2023) stated that the contribution of enslaved African people from the Gold Coast region in Africa, to the technical improvement of gold mining in Minas Gerais, was important for the economic development of both Portugal and colonial Brazil, in the 18th century.

Finally, this study enabled the development a dialogue between mathematics teachers and gold mine owners and guides, which helped them to understand how the use of local mathematical knowledge can enhance, improve, and humanize school mathematical knowledge through the elaboration of ethnomodels in the ethnomodelling process. Thus, the main goal of this study was to explore the contributions of African mathematical knowledge to develop a pedagogical action for ethnomodelling to offer innovative insights and new findings to add to the existing body of knowledge in this research field.

Declarations

Competing Interests The authors declare no competing interests.

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